

# Seminar Logic and Foundations of Computing

## Homework 1

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**Problem 1.** Let  $\mathcal{D}$  be a nonempty small category.

- (a) (5 pt) Show, using the Yoneda Lemma, that for every  $d \in \text{obj } \mathcal{D}$  the colimit of representables  $\mathcal{D}(d, -)$  is the one-point set.
- (b) (5 pt) Use (a) to show 2  $\implies$  3 of Lemma 2.13:

Let  $F : \mathcal{D}' \rightarrow \mathcal{D}$  be a functor such that it satisfies the finality condition with respect to all representable functors  $\mathcal{D}(d, -)$ . Show that for every object  $d$  of  $\mathcal{D}$ , the slice category  $d \downarrow F$  is connected.

(Compare to Exercises 4 and 5 of IX.3, Final Functors in Categories for the Working Mathematician by Mac Lane.)