## Seminar Logic and Foundations of Computing Homework 4

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Exercise 1. Show the following statements, where part (c) assumes the usage of Definition 8.6.

(a) (4 points) A full subcategory  $\mathcal{D}$  of an idempotent complete category  $\mathcal{C}$  is idempotent complete if and only if  $\mathcal{D}$  is closed in  $\mathcal{C}$  under retracts up to isomorphism.

(b) (3 points) A category C is idempotent complete if and only if the idempotent completion  $E_{Ic}: C \to IcC$  of C is an equivalence.

(c) (3 points)  $Ic(\mathcal{C}^{\text{op}}) = (Ic\mathcal{C})^{\text{op}}$  (Note: We assume for a category  $\mathcal{C}$  with objects  $A, B \in \mathcal{C}$  that  $A^{op} = A$  and  $\mathcal{C}(A, B) = \mathcal{C}^{op}(B^{op}, A^{op})$ ).