Seminar Logic and Foundations of Computing
Homework 5

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Problem 1. Let $\mathcal{T}$ be an algebraic theory and $\mathcal{A}$ a full subcategory of $\mathrm{Alg}\,\mathcal{T}$ closed in $\mathrm{Alg}\,\mathcal{T}$ under products, subalgebras, regular quotients, and directed unions (as in Theorem 10.22). Consider the following tasks:

(a) (3 points) Define the action of the functor $R : \mathrm{Alg}\,\mathcal{T} \to \mathcal{A}$ on morphisms, given that it acts on objects by $B \mapsto RB$.

(b) (7 points) Prove that R is indeed a functor.

(c) (Bonus, 2 points) Give a sketch of the fact that R is a left adjoint to the inclusion functor, and conclude that $\mathcal{A}$ is a full regular epireflective subcategory.